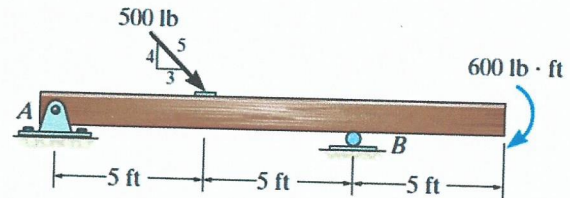


①

1 Determine the horizontal and vertical components of reaction at the supports. Neglect the thickness of the beam.



(Ans.  $A_x = 300 \text{ lb}$ ,  $A_y = 140 \text{ lb}$ ,  
 $B_y = 260 \text{ lb}$ )

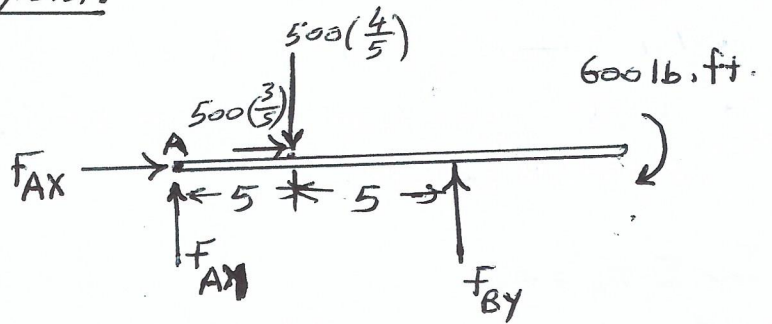
\* I- we draw a free body diagram.

$$\rightarrow \sum F_x = 0$$

$$F_{Ax} + 500\left(\frac{3}{5}\right) = 0$$

$$F_{Ax} = -300 \text{ lb}$$

∴ The resultant force in  $F_{Ax}$  direction = 300 lb in the negative x-direction.



$$\sum M_A = 0 \quad \therefore F_{By}(10) - 500\left(\frac{4}{5}\right)(5) - 600 = 0$$

$$10 F_{By} - 2000 - 600 = 0$$

$$\therefore 10 F_{By} = 2600$$

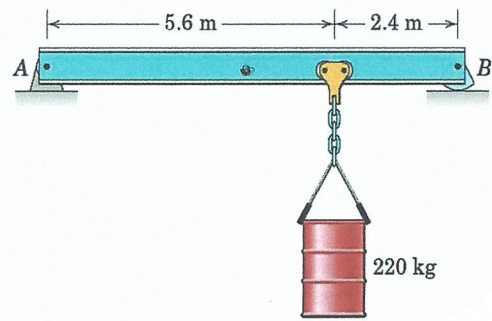
$$\therefore F_{By} = 260 \text{ lb.}$$

∴  $F_{By} = 260 \text{ lb}$ , its direction is in the +ve y direction.

$$+\uparrow \sum F_y = 0 \quad \therefore F_{Ay} - 500\left(\frac{4}{5}\right) + 260 = 0$$

$$\therefore F_{Ay} = 400 - 260 = 140 \text{ lb.}$$

2 Determine the reactions at the supports at A and B, and the tension in the cable. The I-beam is uniform with weight = 450 kg.  
 (Ans.  $A_x = 0 \text{ N}$ ,  $A_y = 2850 \text{ N}$ ,  $B_y = 3720 \text{ N}$ )



- Pin support at A then we have  $A_x, A_y$
- roller support at B. then we have force  $\perp$  the surface at the point of contact.

$$\rightarrow \sum F_x = 0$$

$$\therefore A_x = 0$$

$$\uparrow \sum F_y = 0$$

$$B_y + A_y - (220 \times 9.81) - 450 \times 9.81 = 0$$

$$\therefore A_y + B_y = 6572.7 \quad \text{--- (1)}$$

$$\sum M_A = 0$$

$$B_y (8) - (220 \times 9.81) \times 5.6 - (450 \times 9.81) \times 4 = 0$$

$$8B_y = 29743.92$$

$$\therefore B_y = 3718 \text{ N.}$$

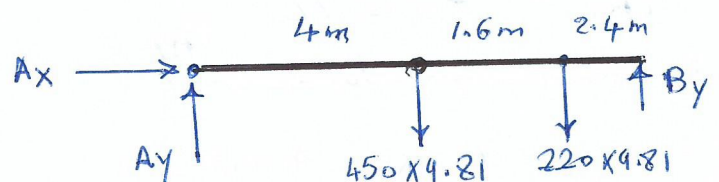
$$\therefore A_y = 6572.7 - 3718$$

$$A_y = 2854.7 \text{ N.}$$

$$A_x = 0$$

Tension in the cable = ??

$$T = 220 \times 9.81 = 2158 \text{ N.}$$

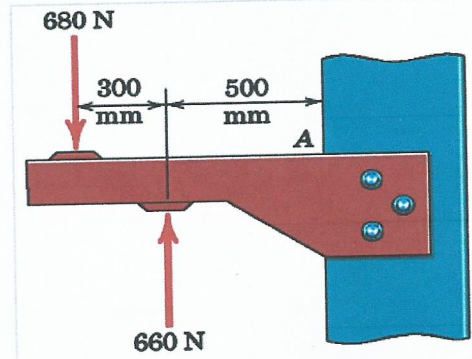


3

3

3 Determine the reactions at the support A.

(Ans.  $A_x = 0\text{ N}$ ,  $A_y = 20\text{ N}$ ,  $M_A = -214\text{ N.m}$ )



$$\rightarrow \sum F_x = 0$$

$$A_x = 0\text{ N}$$

$$\uparrow \sum F_y = 0$$

$$A_y + 660 - 680 = 0$$

$$A_y = 680 - 660 = 20\text{ N}$$

$$\sum M_B = 0$$

$$660 \times 0.3\text{ m} + A_y (0.8\text{ m}) + M_A = 0$$

$$(660 \times 0.3) + 20(0.8) + M_A = 0$$

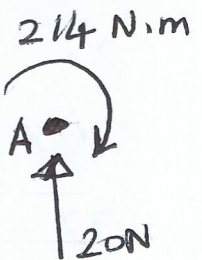
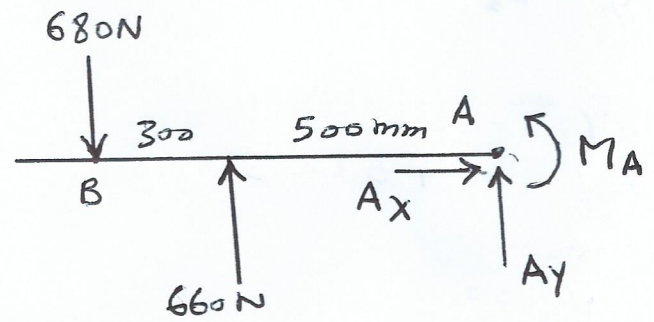
$$M_A = -20(0.8) - 660(0.3)$$

$$M_A = -214\text{ N.m}$$

At support A we have 2 reactions:

① Force = 20 N in the positive direction of Y.

② Moment = 214 N.m in the C.W direction.



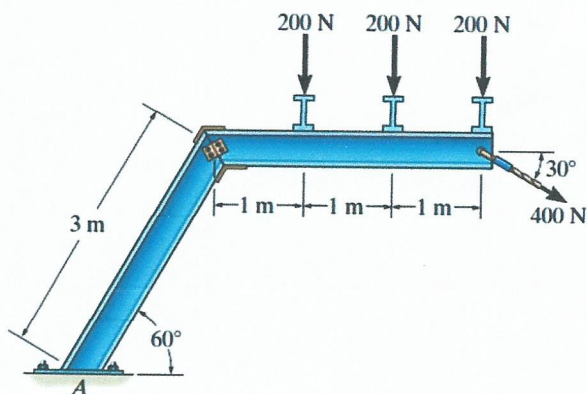


4

4

4 Determine the components of reaction at the fixed support A. Neglect the thickness of the beam.

(Ans.  $A_x = 346 \text{ N}$ ,  $A_y = 800 \text{ N}$ ,  
 $M_A = 3900 \text{ N.m}$ )

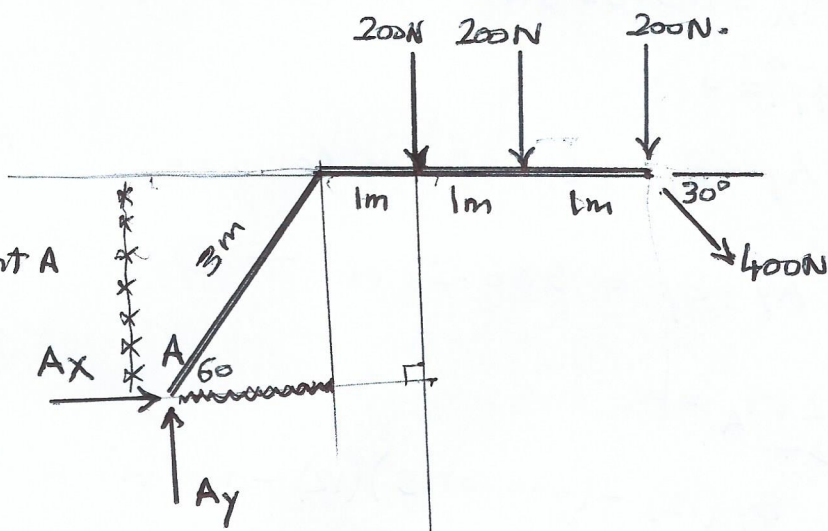


$$\rightarrow \sum F_x = 0$$

$$A_x + 400 \cos 30 = 0$$

$$\therefore A_x = -346 \text{ N.}$$

$\therefore$  the x-component at point A is equal to 346 N and acts in the -ve direction of x-axis.



$$\uparrow \sum F_y = 0$$

$$A_y - 200 - 200 - 200 - 400 \sin 30 = 0$$

$$A_y = 800 \text{ N}$$

$\therefore$  The y-component at point A is equal to 800 N and acts in the +ve direction of y.

$$\begin{aligned} \overset{+}{\curvearrowright} M_A &= -200(1 + 3 \cos 60) - 200(2 + 3 \cos 60) - 200(3 + 3 \cos 60) \\ &\quad - 400 \cos 30 (3 \sin 60) - 400 \sin 30 (3 + 3 \cos 60) \end{aligned}$$

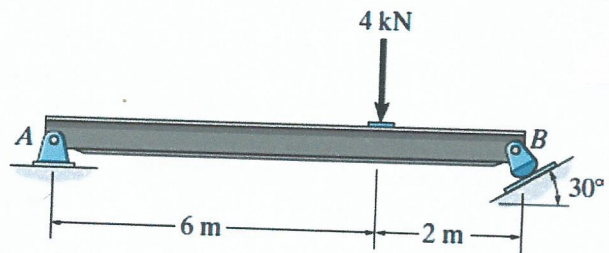
$$\overset{+}{\curvearrowright} M_A = -500 - 700 - 900 - 900 - 900$$

$$M_A = -3900 \text{ N.m} \quad \therefore M_A = 3900 \text{ N.m C.W around A.}$$

5

5 Determine the horizontal and vertical components of reaction at the pin A and the reaction of the rocker B on the beam.

(Ans.  $A_x = 1.73 \text{ kN}$ ,  $A_y = 1 \text{ kN}$ ,  
 $N_B = 3.46 \text{ kN}$ )



$$\rightarrow \sum F_x = 0$$

$$A_x - B \sin 30 = 0$$

$$\uparrow \sum F_y = 0$$

$$A_y - 4 + B \cos 30 = 0$$

$$\sum M_A = 0$$

$$B \cos 30 (8) - 4 (6) = 0$$

$$8B \frac{\sqrt{3}}{2} - 24 = 0 \quad \Rightarrow \quad \frac{8B\sqrt{3}}{2} = 24 \quad \Rightarrow \quad 4B\sqrt{3} = 24$$

$$\Rightarrow B\sqrt{3} = 6 \quad \Rightarrow \quad B = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3} = 3.46 \text{ kN}$$

$$A_x = B \sin 30$$

$$\Rightarrow A_x = 3.46 \left( \frac{1}{2} \right) \text{ kN}$$

$$\boxed{A_x = 1.73 \text{ kN}}$$

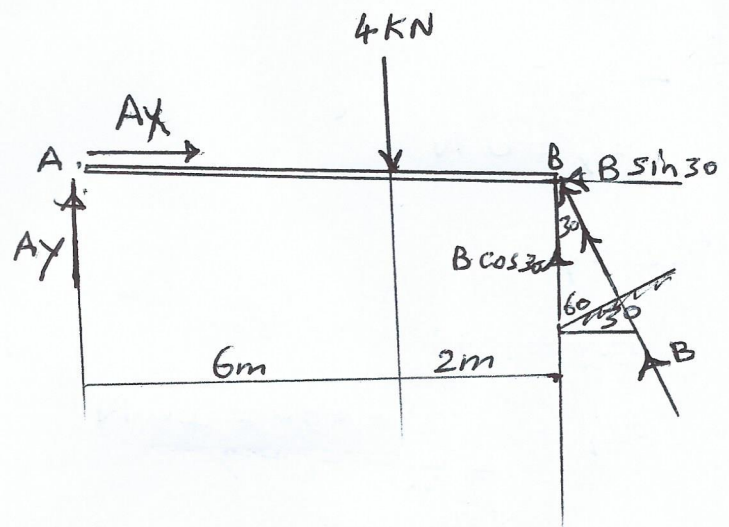
$$A_y = 4 - B \cos 30$$

$$A_y = 4 - 3.46 \left( \frac{\sqrt{3}}{2} \right) = 4 - 3 = 1 \text{ kN}$$

$$\boxed{A_y = 1 \text{ kN}}$$

reaction of the rocker B on the beam =  $B = 3.46$

$$\boxed{\text{reaction on B} = 3.46 \text{ kN}}$$

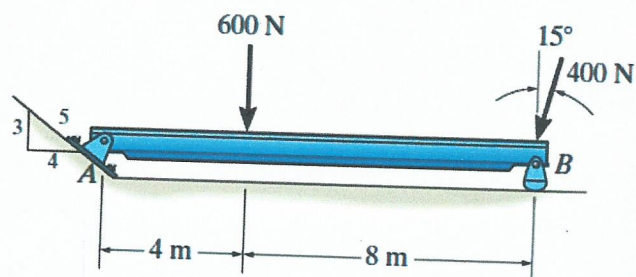




⑥

6 Determine the magnitude of the reactions on the beam at A and B. Neglect the thickness of the beam.

(Ans.  $B_y = 586 \text{ N}$ ,  $F_A = 413 \text{ N}$ )



$$\rightarrow \sum F_x = 0$$

$$A_x - 400 \sin 15 = 0$$

$$A_x = 103.528 \text{ N} \quad \text{--- (1)}$$

$$\sum F_y \uparrow = 0$$

$$A_y + B_y - 600 - 400 \cos 15 = 0$$

$$A_y + B_y = 986.37 \text{ N} \quad \text{--- (2)}$$

$$\sum M_A = 0$$

$$B_y (12) - (400 \cos 15)(12) - 600(4) = 0 \quad \text{--- (3)}$$

From (3) get  $B_y$

$$12B_y = 7036.44$$

$$\boxed{B_y = 586 \text{ N}}$$

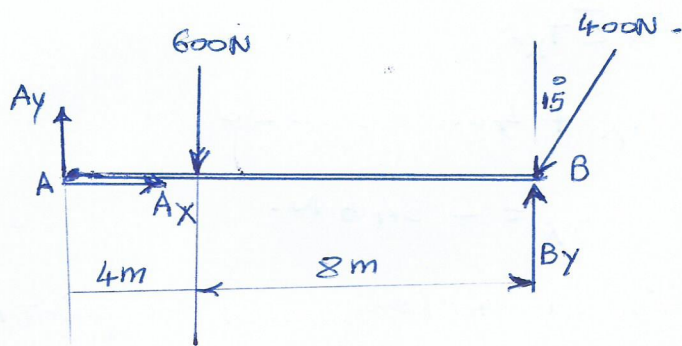
From (2)

$$\therefore A_y = 986.37 - 586 = 400.37 \text{ N}$$

$$\boxed{A_y = 400.37 \text{ N}}$$

$$F_A = \sqrt{(103.528)^2 + (400.37)^2} = 413.54 \text{ N}$$

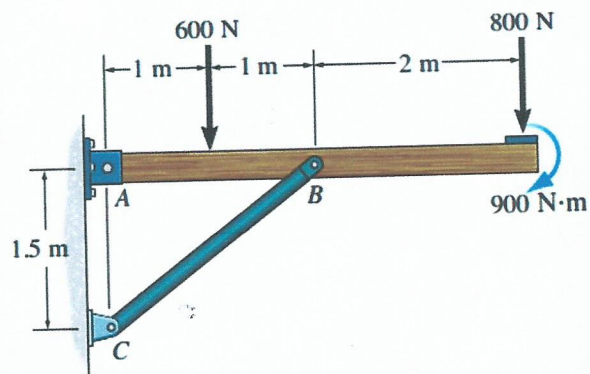
$$\boxed{F_A = 413.5 \text{ N}}$$



7

7 The overhanging beam is supported by a pin at A and the two-force strut BC. Determine the horizontal and vertical components of reaction at A and the reaction at B on the beam.

(Ans.  $A_x = 3133.33 \text{ N}$ ,  $A_y = 950 \text{ N}$ ,  $F_{BC} = 3916.67 \text{ N}$ )



$$\sum F_x = 0$$

$$A_x - F_{BC} \cos \theta = 0$$

but from Geometry

$$\tan \theta = \frac{1.5}{2}$$

$$A_x = F_{BC} (0.8) \quad \text{--- (1)}$$

$$A_x =$$

$$\sum F_y = 0$$

$$A_y - 600 - F_{BC} \sin \theta - 800 = 0$$

$$A_y - 1400 - F_{BC} (0.6) = 0 \quad \text{--- (2)}$$

$$\sum M_A = 0$$

$$-900 - 600(1) - F_{BC}(1.2) - 800(4) - 900 = 0$$

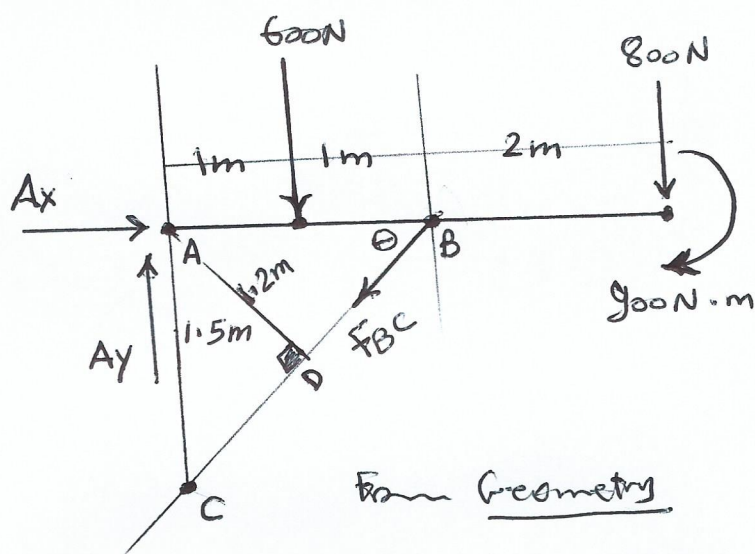
$$-4700 - 1.2 F_{BC} = 0 \quad \text{--- } 1.2 F_{BC} = -4700$$

$$F_{BC} = -3916.67 \text{ N}$$

$$A_x = 0.8 F_{BC} = -3133.33 \text{ N}$$

$$A_y = 1400 + 0.6 F_{BC}$$

$$A_y = 1400 - 2350 = 950 \text{ N}$$



From Geometry

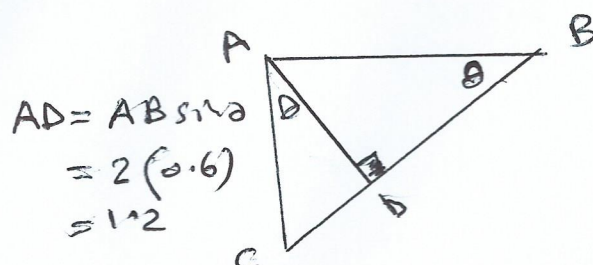
$$BC^2 = 4 + 2.25$$

$$BC^2 = 6.25$$

$$BC = 2.5 \text{ m}$$

$$\sin \theta = \frac{1.5}{2.5} = 0.6$$

$$\cos \theta = \frac{2}{2.5} = 0.8$$

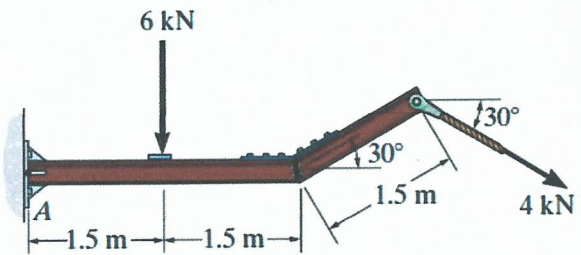




8

8

8 Determine the components of the support reactions at the fixed support A on the cantilevered beam.



(Ans.  $A_x = 3.46 \text{ kN}$ ,  $A_y = 8 \text{ kN}$ ,  $M_A = 20.2 \text{ kN.m}$ )

Fixed support at A  
we have  $A_x, A_y, M_A$

$$\sum F_x = 0$$

$$A_x + 4 \cos 30 = 0$$

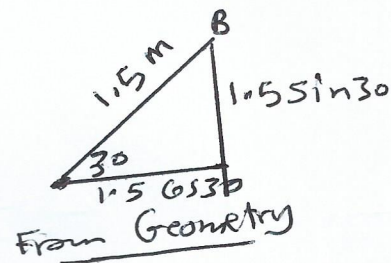
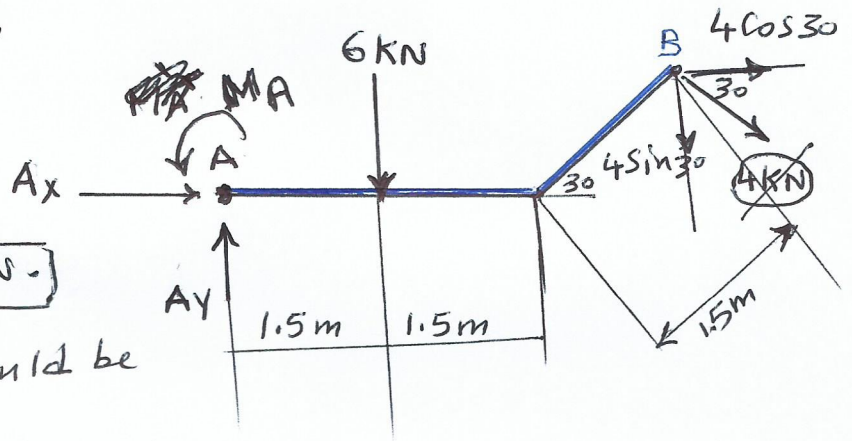
$$\therefore A_x = -\frac{4\sqrt{3}}{2} = \boxed{-3.46 \text{ kN}}$$

$\therefore$  The direction of  $A_x$  should be in the -ve x-direction.

$$\sum F_y = 0$$

$$A_y - 6 - 4 \sin 30 = 0$$

$$A_y = 6 + 4\left(\frac{1}{2}\right) \quad \therefore \boxed{A_y = 8 \text{ kN}}$$



to find  $M_A$ .

we will take the moment about A:

$$M_A - 6(1.5) - 4 \sin 30 (1.5 \cos 30 + 3) - 4 \cos 30 (1.5 \sin 30) = 0$$

$$M_A - 9 - 8.598 - 2.598 = 0$$

$$M_A = 20.2 \text{ kN.m}$$

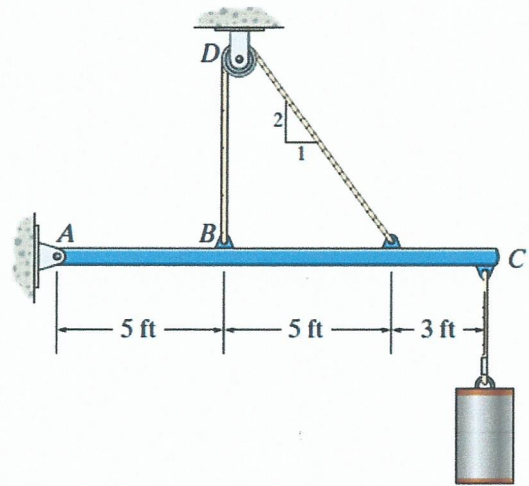


9

9

9 Determine the tension in the cable and the horizontal and vertical components of reaction of the pin A. The pulley at D is frictionless and the cylinder weighs 80 lb.

(Ans.  $A_x = 33.4 \text{ lb}$ ,  $A_y = 61.3 \text{ lb}$ ,  $T = 74.6 \text{ lb}$ )



• Draw Free body diagram.

$$\rightarrow \sum F_x = 0$$

$$A_x - T \left( \frac{1}{\sqrt{5}} \right) = 0 \quad \text{--- (1)}$$

$$\uparrow \sum F_y = 0$$

$$T + T \left( \frac{2}{\sqrt{5}} \right) - 80 - A_y = 0 \quad \text{--- (2)}$$

$$\left( \sum M_A = 0 \right)$$

$$T(5) + T \left( \frac{2}{\sqrt{5}} \right) (10) - 80(13) = 0 \quad \text{--- (3)}$$

From (3)

$$5T + 8.94T - 1040 = 0 \quad \Rightarrow T = 74.6 \text{ lb}$$

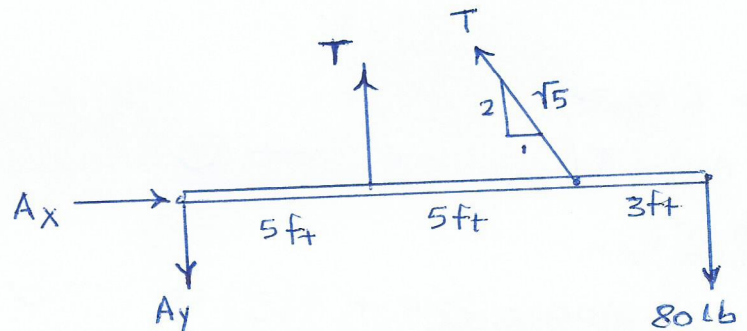
From (1)

$$A_x = \frac{T}{\sqrt{5}} = 33.4 \text{ lb}$$

$$A_x = 33.4 \text{ lb}$$

From (2)

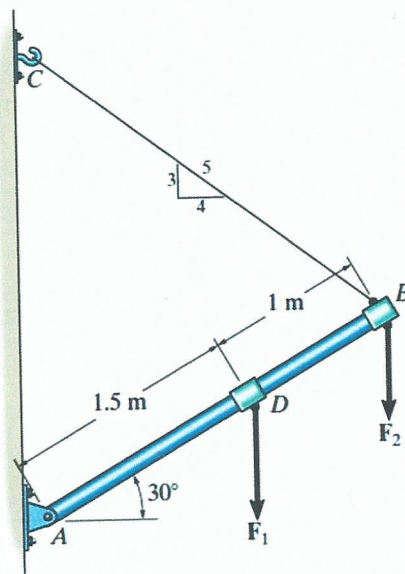
$$A_y = 61.3 \text{ lb}$$



(10)

10 The boom supports the two vertical loads. Neglect the size of the collars at D and B and the thickness of the boom, and compute the horizontal and vertical components of force at the pin A and the force in cable CB. Set  $F_1 = 800 \text{ N}$  and  $F_2 = 350 \text{ N}$ .

(Ans.  $A_x = 625 \text{ N}, A_y = 681 \text{ N}, F_{CB} = 782 \text{ N}$ )



$$+\rightarrow \sum F_x = 0$$

$$A_x - T\left(\frac{4}{5}\right) = 0 \quad \text{--- (1)}$$

$$+\uparrow \sum F_y = 0$$

$$A_y - 800 - 350 + T\left(\frac{3}{5}\right) = 0 \quad \text{--- (2)}$$

$$\left( \sum M_A = 0 \right)$$

$$-350(2.5)\left(\frac{4}{5}\right) - (800)(1.5)(\cos 30^\circ)$$

$$-800(1.5)(\cos 30^\circ)$$

$$+ T\left(\frac{4}{5}\right)(2.5 \sin 30^\circ) + \left(\frac{3}{5}\right)T(2.5 \cos 30^\circ) = 0 \quad \text{--- (3)}$$

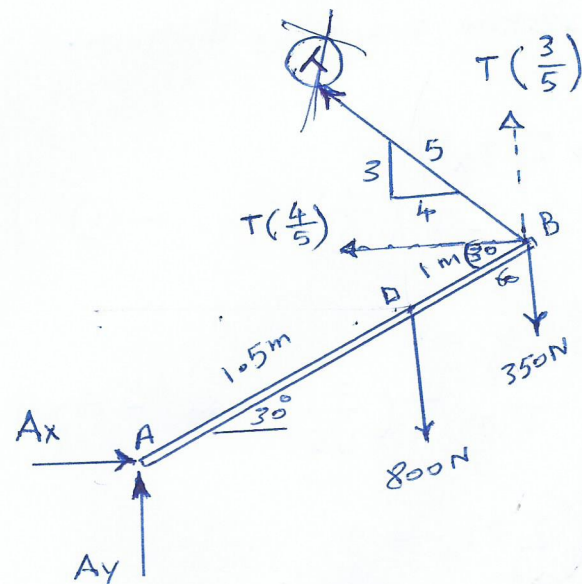
From (3) we can get T

$$-757.772 - 1039.23 + T + 1.299T = 0$$

$$2.299T = 1797 \quad \therefore \boxed{T = 782 \text{ N}}$$

From (1)  $A_x = \frac{4}{5}(T) = 625 \text{ N} \quad \therefore \boxed{A_x = 625 \text{ N}}$

From (2)  $A_y - 1150 + \frac{3}{5}(T) = 0$   
 $\therefore A_y = 1150 - 469.2 \quad \therefore \boxed{A_y = 681 \text{ N}}$



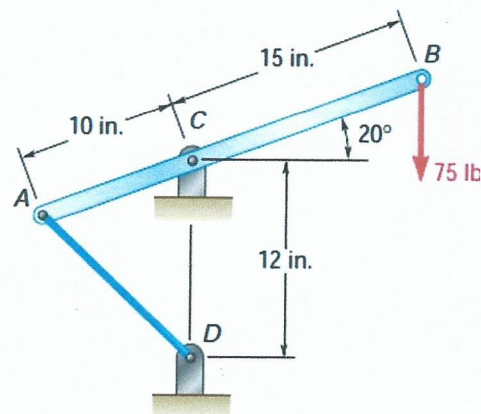


11

11

11 A lever AB is hinged at C and attached to a control cable at A. If the lever is subjected to a 75-lb vertical force at B, determine (a) the tension in the cable, (b) the reaction at C.

(Ans.  $C_x = -88.097 \text{ lb}$ ,  $C_y = 155.435 \text{ lb}$ ,  $C = 178.665 \text{ lb}$ ,  $T_{AB} = 119.293 \text{ lb}$ )



$$+\rightarrow \Sigma F_x = 0$$

$$C_x + T_{AD} \cos \alpha = 0 \quad \text{--- (1)}$$

$$+\uparrow \Sigma F_y = 0$$

$$C_y - 75 - T_{AD} \sin \alpha = 0 \quad \text{--- (2)}$$

$$\Sigma M_C = 0$$

$$-75 \times 15 (\cos 20) + T_{AD} \sin \alpha (10 \cos 20)$$

$$+ T_{AD} \cos \alpha (10 \sin 20) \quad \text{--- (3)}$$

From (3) if we have  $\alpha$  we can get  $T_{AD}$

\* From Geometry

$$CE = 10 \sin 20 = 3.42 \text{ in.}$$

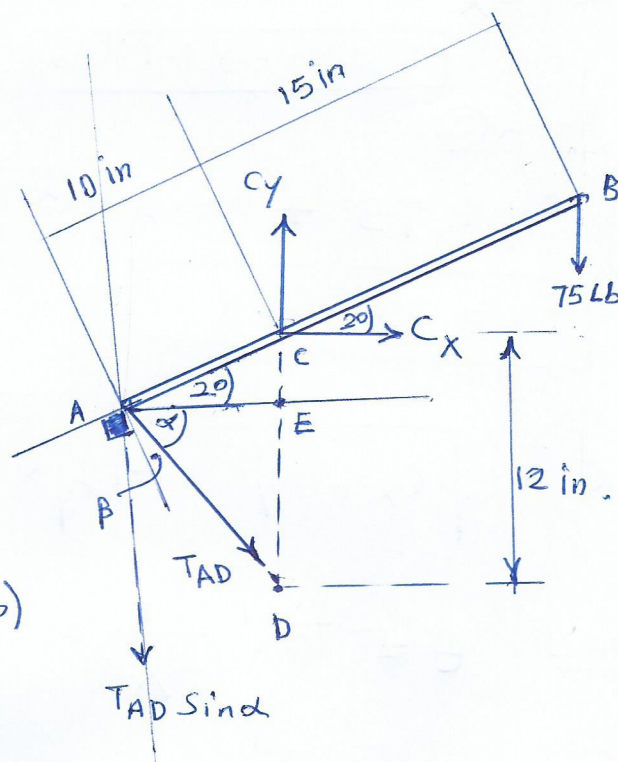
$$AE = 10 \cos 20 = 9.397 \text{ in.}$$

$$DE = 12 - CE = 12 - 3.42 = 8.58 \text{ in.}$$

$$\tan \alpha = \frac{DE}{AE} = \frac{8.58}{9.397}$$

$$\alpha = \tan^{-1}(0.913)$$

$$\alpha = 42.396^\circ$$



From ③

$$-1057.154 + 9.397 T_{AD} * 0.674 + 3.42 T_{AD} * 0.739 = 0$$

~~1057.154~~

$$6.334 T_D + 2.527 T_D = 1057.154$$

$$\therefore T_D = \frac{1057.154}{8.861} = 119.3 \text{ lb}$$

$$\boxed{T_D = 119.3 \text{ lb}}$$

From ①

$$\therefore C_x = -119.3 (\cos 42.396) = -88.1 \text{ lb}$$

$$\boxed{C_x = -88.1 \text{ lb}}$$


From ②

$$C_y = 75 + 119.3 (\sin 42.396)$$

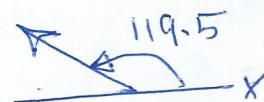
$$\boxed{C_y = 155.438 \text{ lb}}$$

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-88.1)^2 + (155.438)^2} = 178.67 \text{ lb}$$

and  $\theta = \tan^{-1} \frac{C_y}{C_x} = \frac{155.438}{88.1} = 60.5^\circ$

$\therefore$  The resultant of C is 178.67   $60.5^\circ$

⑤ 178.67 lb and angle 119.5 with the x-axis.

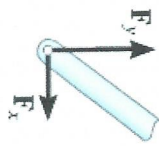
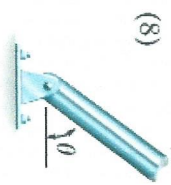




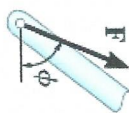
## Types of Connection

## Reaction

## Number of Unknowns



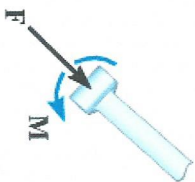
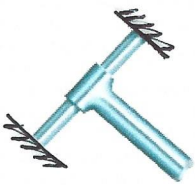
or



Two unknowns. The reactions are two components of force, or the magnitude and direction  $\phi$  of the resultant force. Note that  $\phi$  and  $\theta$  are not necessarily equal [usually not, unless the rod shown is a link as in (2)].

smooth pin or hinge

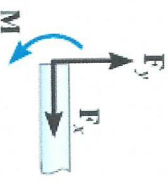
(9)



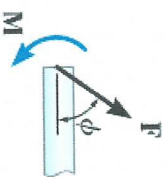
Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.

member fixed connected to collar on smooth rod

(10)



or



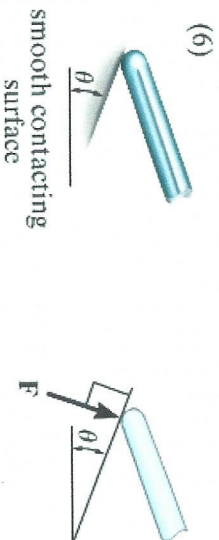
fixed support

Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction  $\phi$  of the resultant force.

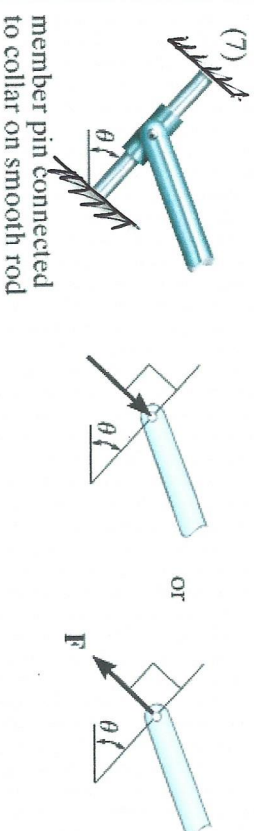




One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.



One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.





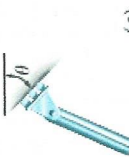
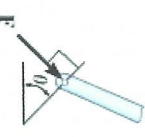




One unknown. The reaction is a force which acts perpendicular to the rod.

*continued*



**TABLE 5-1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems**

Types of Connection	Reaction	Number of Unknowns
(1) cable 		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2) weightless link 		One unknown. The reaction is a force which acts along the axis of the link.
(3) roller 		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4) roller or pin in confined smooth slot 		One unknown. The reaction is a force which acts perpendicular to the slot.